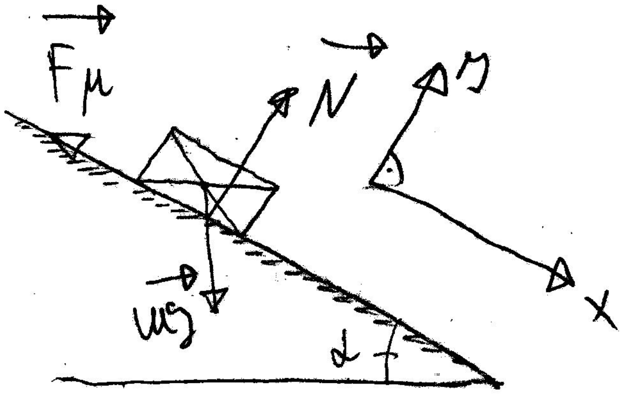
3.4

$\alpha = 25^\circ$

$\Delta t = 4s$

$\mu = 0,04$

$V_1 - ?$



$$\vec{K}_1 - \vec{K}_0 = \int_0^4 \vec{F}\, dt$$

$$x:\ mV_1 - m\underbrace{V_0}_{0} = \int_0^4 (mg\sin\alpha - mg\cos\alpha\,\mu)\, dt$$

$$mV_1 = mg(\sin\alpha - \mu\cos\alpha)\Delta t \quad |:m$$

$$V_1 = g(\sin\alpha - \mu\cos\alpha)\Delta t$$

$$V_1 = 9{,}81(\sin 25^\circ - 0{,}04\cos 25^\circ)\cdot 4$$

$$\boxed{V_1 = 15{,}161\ m/s}$$

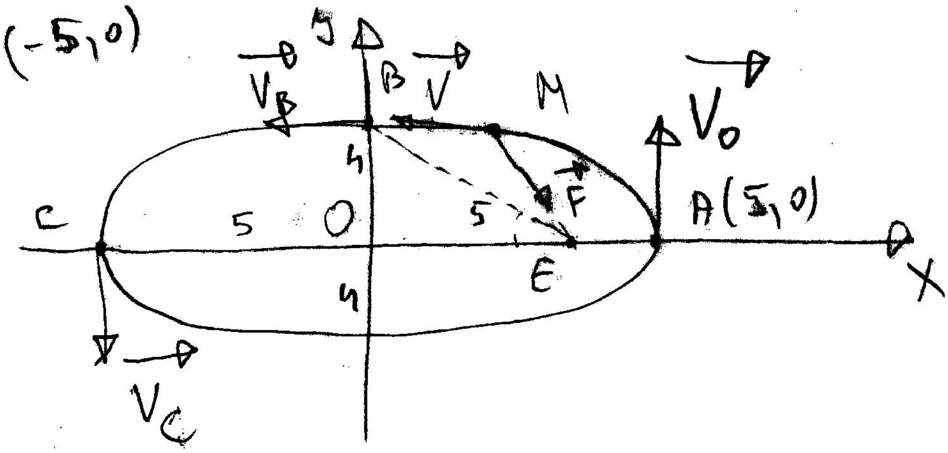
3.7

$\frac{x^2}{25}+\frac{y^2}{16}=1$ B(0,4) C(-5,0)

A(5,0)

$\vec{V}_0 = 2\vec{j}$

$\vec{V}_B,\ \vec{V}_C - ?$



$\vec{L}_E = const.$

$\vec{r}\times m\vec{V} = \vec{r}_0 \times m\vec{V}_0$

$\vec{r}_B \times m\vec{V}_B = \vec{r}_0 \times m\vec{V}_0$

z: $\overline{EB}\,\cancel{m}\,V_B \sin\angle(\vec{r}_B,\vec{V}_B) = \overline{EA}\,\cancel{m}\,V_0 \sin\angle(\vec{r}_0,\vec{V}_0)$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

$\overline{OE}=ae=5\,\frac{3}{5}=3$, $\overline{EA}=a-\overline{OE}=5-3=2$

$\sin\angle(\vec{r}_B,\vec{V}_B)=\frac{b}{\overline{EB}}$; $\sin\angle(\vec{r}_0,\vec{V}_0)=1$

$\cancel{\overline{EB}}\,V_B\,\frac{b}{\cancel{\overline{EB}}}=\overline{EA}\,V_0\cdot 1$

$bV_B=\overline{EA}\,V_0 \Rightarrow V_B=\frac{\overline{EA}}{b}V_0=\frac{2}{4}\,2=1$, $\vec{V}_B=-\vec{i}$

$\vec{r}_C\times m\vec{V}_C=\vec{r}_0\times m\vec{V}_0$

z: $\overline{EC}\,\cancel{m}\,V_C \underbrace{\sin\angle(\vec{r}_C,\vec{V}_C)}_{=1} = \overline{EA}\,\cancel{m}\,V_0 \underbrace{\sin\angle(\vec{r}_0,\vec{V}_0)}_{=1}$

$(\overline{OE}+a)V_C=\overline{EA}\,V_0 \Rightarrow V_C=\frac{\overline{EA}}{a+\overline{OE}}V_0=\frac{2}{8}\,2=\frac{1}{2}$

$$\vec{V}_C = -\frac{1}{2}\vec{j}$$

$$\vec{V}_D \times \omega \vec{V}_D = \omega \vec{V}_D \times \vec{V}_D = \omega \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} =$$

$$= 4\omega \vec{K}$$

$$\vec{V}_B \times \omega \vec{V}_B = \omega \vec{V}_B \times \vec{V}_B = \omega \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 4 & 0 \\ V_{BX} & 0 & 0 \end{vmatrix} =$$

$$= -4\omega V_{BX}\vec{K}$$

$$4\omega \vec{K} = -4\omega V_{BX}\vec{K} \Rightarrow \boxed{V_{BX} = -1}$$

$$\vec{V}_C \times \omega \vec{V}_C = \omega \vec{V}_C \times \vec{V}_C = \omega \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & 0 & 0 \\ 0 & V_{Cy} & 0 \end{vmatrix} =$$

$$= -8\omega V_{Cy}\vec{K}$$

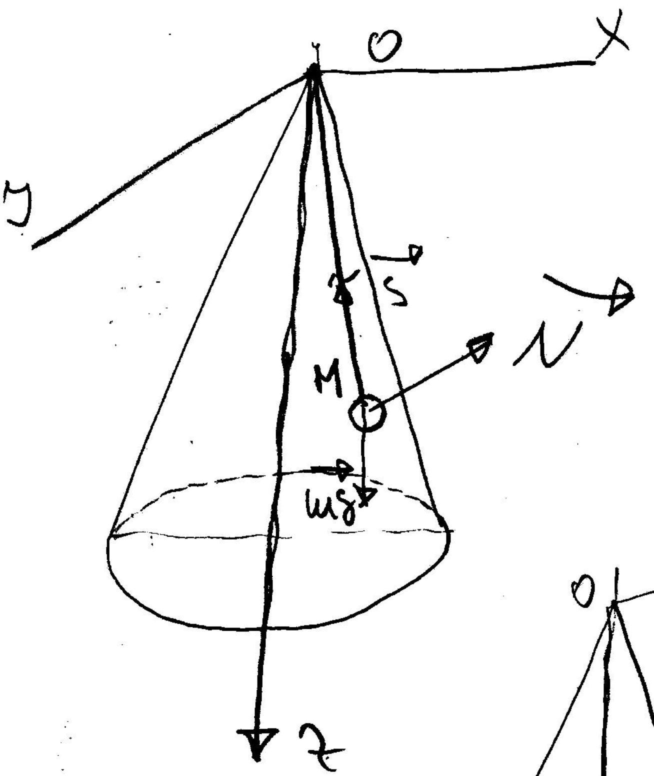
$$4\omega \vec{K} = -8\omega V_{Cy}\vec{K} \Rightarrow \boxed{V_{Cy} = -\frac{1}{2}}$$

3.9 $u = const.$

$t_0 = 0$

v_0, r_0

$v(t)-?$



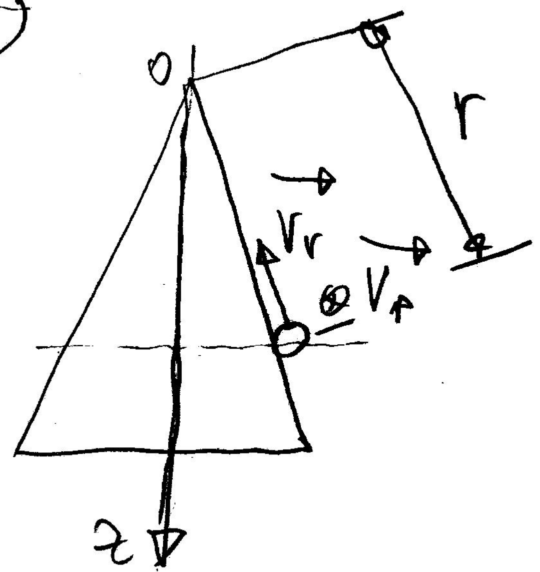
$L_{0z} = const.$

$L_{0z}(t_0) = L_{0z}(t)$

$r = r_0 - ut$

$\vec{L}_0 = \vec{r} \times m\vec{v}$

$\vec{L}_0(t_0) = \vec{r}_0 \times m\vec{v}_0$



$\vec{v} = \vec{v}_r + \vec{v}_p$

$v_r = u$

$v_0^2 = v_{p0}^2 + u^2 \Rightarrow v_{p0} = \sqrt{v_0^2 - u^2}$

$v^2 = v_p^2 + u^2 \Rightarrow v_p = \sqrt{v^2 - u^2}$

$\vec{L}_0(t) = \vec{r} \times m(\vec{v}_p + \vec{v}_r) = \vec{r} \times m\vec{v}_p + \underbrace{\vec{r} \times m\vec{v}_r}_{0}$

$= \vec{r} \times m\vec{v}_p$

$\vec{L}_0(t_0) = \vec{r}_0 \times m\vec{v}_{p0}$

$L_{0z}(t) = m r v_p \sin\angle(\vec{r}, \vec{v}_p) = m r v_p$

$L_{0z}(t_0) = m r_0 v_{p0} \sin\angle(\vec{r}_0, \vec{v}_{p0}) = m r_0 v_{p0}$

$$\cancel{m}\, r_0 V_{\varphi 0} = \cancel{m}\, r V_\varphi$$

$$r_0 V_{\varphi 0} = r V_\varphi$$

$$r_0 \sqrt{V_0^2 - u^2} = r \sqrt{V^2 - u^2} \,\Big|^2$$

$$r_0^2 (V_0^2 - u^2) = r^2 (V^2 - u^2)$$

$$V^2 = u^2 + \frac{r_0^2}{r^2} (V_0^2 - u^2)$$

$$V = \sqrt{u^2 + \frac{r_0^2 (V_0^2 - u^2)}{(r_0 - ut)^2}} =$$

$$= \sqrt{\frac{u^2(\cancel{r_0^2} - 2r_0 ut + u^2 t^2) + r_0^2 V_0^2 - \cancel{r_0^2 u^2}}{(r_0 - ut)^2}} =$$

$$= \frac{\sqrt{r_0^2 V_0^2 - 2 r_0 u^3 t + u^4 t^2}}{r_0 - ut}$$

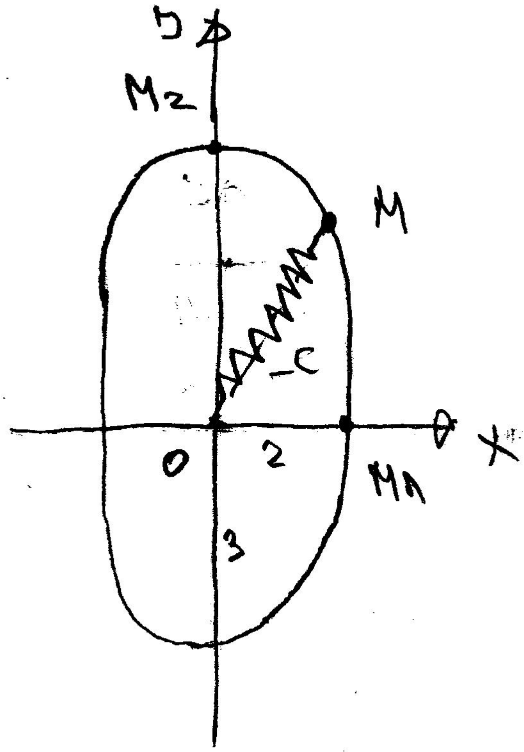
3.13

$c = 2\ N/m$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$l_0 = 1 m$

$A_{12} - ?$

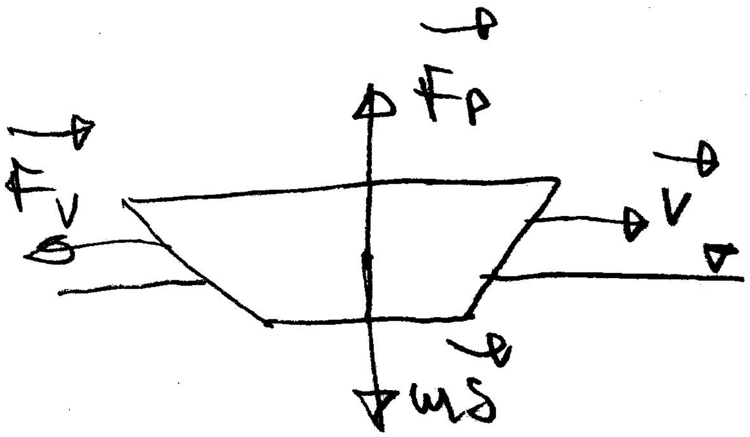


$$A_{12} = \frac{c}{2}\left[(r_1 - l_0)^2 - (r_2 - l_0)^2\right] =$$
$$= \frac{2}{2}\left[(2-1)^2 - (3-1)^2\right] = 1(1-4) = -3\ J$$

3.20

$V_2 = 2V_1$

$F_v = bV^2$



$P_1 = F_{v1} V_1 =$

$= bV_1^2 V_1 = bV_1^3$

$P_2 = F_{v2} V_2 = b(2V_1)^2\, 2V_1 = 8bV_1^3$

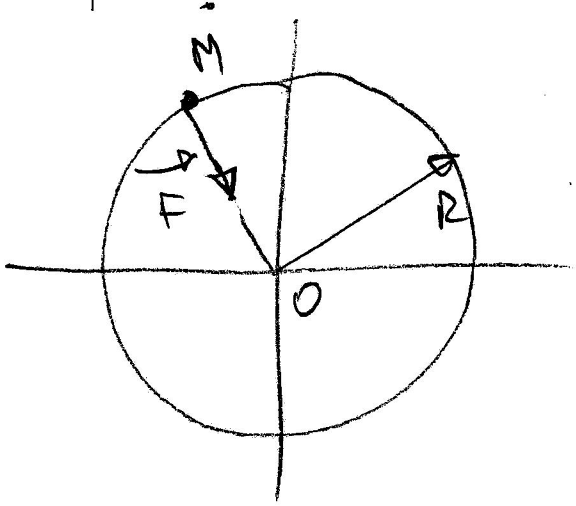
$\frac{P_2}{P_1} = \frac{8bV_1^3}{bV_1^3} = 8$

3.22

R

F

$T - ?$



$$T = \frac{1}{2} m V^2$$

$$m\vec{a} = \vec{F}$$

$$n = m\frac{V^2}{R} = F$$

$$V^2 = \frac{FR}{m}$$

$$T = \frac{1}{2}\cancel{m}\,\frac{FR}{\cancel{m}} = \frac{1}{2}FR$$

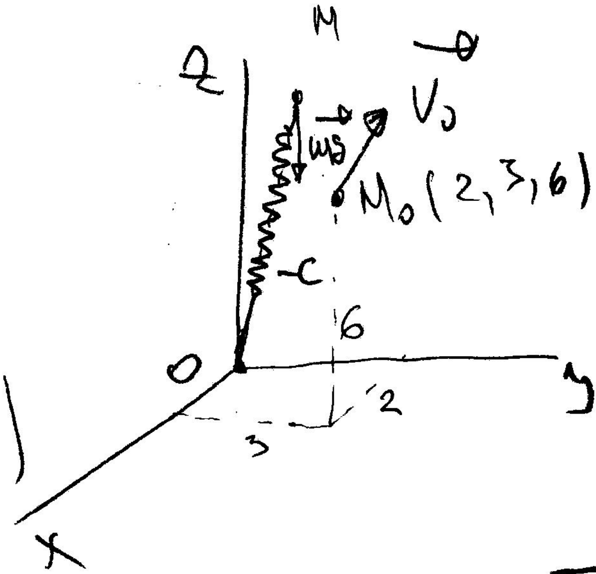
3.25

m

c

$M_0(2,3,6)$

V_0



$$T_1 - T_0 = A(m\vec{g}) + A(\vec{F_c})$$

$$\Big| \cdot \frac{2}{m}$$

$$\frac{1}{2}mV_1^2 - \frac{1}{2}mV_0^2 = -mg(z-6) - \frac{c}{2}\left(\sqrt{x^2+y^2+z^2} - \sqrt{(2^2+3^2+6^2)}\right)^2$$

$$V_1^2 = V_0^2 - 2g(z-6) - \frac{c}{m}\left(\sqrt{x^2+y^2+z^2} - 7\right)^2$$

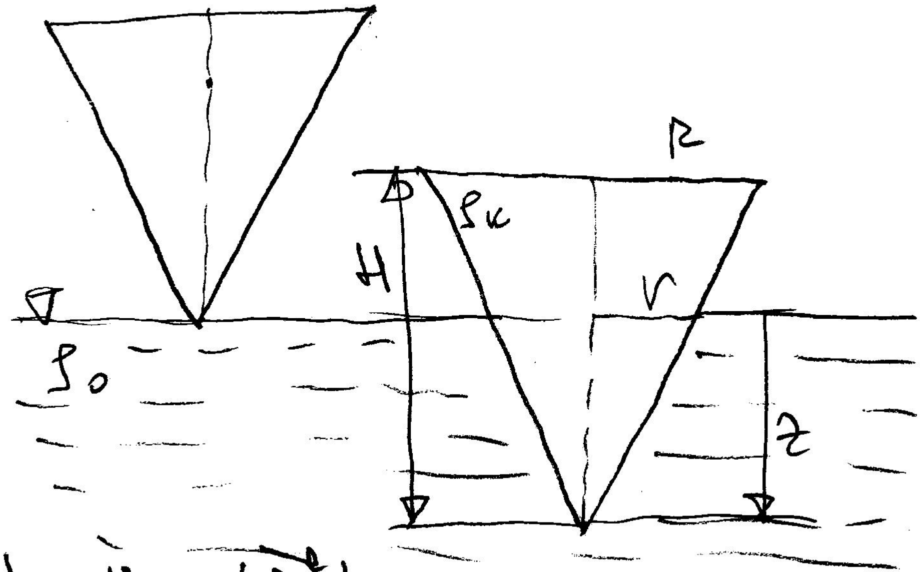
3.27

$v_0 = 0$

H, R

$\rho_k = 2\rho_v$

$v_1 - ?$



$T_1 - T_0 = A_{0,1}(m\vec{g}) + A_{0,1}(\vec{F_p})$ ($T_0 = 0$)

$\frac{1}{2} m v_1^2 = A_{0,1}(m\vec{g}) + A_{0,1}(\vec{F_p})$

$A_{0,1}(m\vec{g}) = + m g H$

$A_{0,1}(\vec{F_p}) = -\int_0^H F_p \, dz \,, \quad F_p = \rho_v g V$

$\frac{R}{H} = \frac{r}{z}$

$r = \frac{R}{H} z$

$V = \frac{1}{3} r^2 \pi z = \frac{1}{3} \frac{R^2}{H^2} \pi z^3$

$A_{0,1}(\vec{F_p}) = -\int_0^H \rho_v \frac{1}{3} \frac{R^2}{H^2} g \pi z^3 \, dz =$

$= -\frac{1}{3} \frac{\rho_v R^2 g}{H^2} \pi \left. \frac{z^4}{4} \right|_0^H = -\frac{\rho_v g}{12} \frac{R^2 \pi}{H^2} H^4 = -\frac{\rho_v g}{12} R^2 \pi H^2$

$m = \rho_k V = 2\rho_v \frac{1}{3} R^2 \pi H$

$\frac{1}{2} m v_1^2 = m g H - \frac{\rho_v g}{12} R^2 \pi H^2 \quad / \cdot \frac{2}{m}$

$$V_1^2 = 2gH - \frac{\cancel{2}}{\cancel{\frac{2}{3}\rho_0 R^2 \pi H}} \cdot \frac{\cancel{\rho_0} g}{\cancel{12}_4} \cancel{R^2 \pi} H^2 =$$

$$= 2gH - \frac{gH}{4} = \frac{7gH}{4}$$

$$\boxed{V_1 = \frac{1}{2}\sqrt{7gH}}$$

3.28

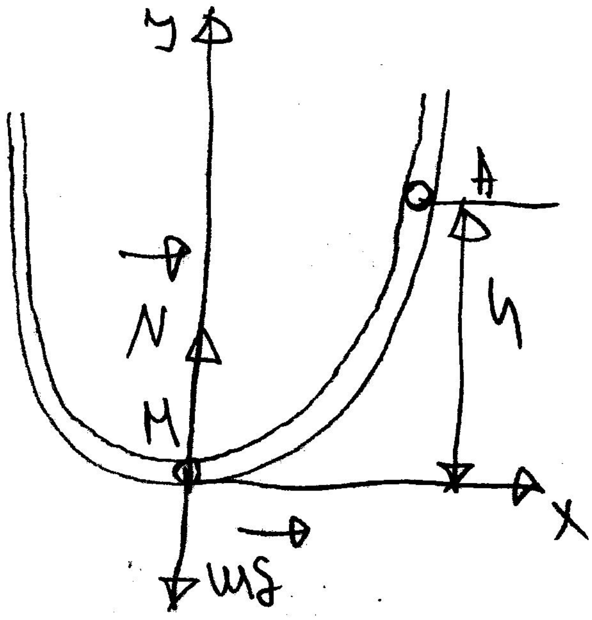
$y = x^2$

$A(\sqrt{2}, 2)$

$V_0 = 0$

$m = 0{,}01 kg$

$N_1 - ?$



$$T_1 - \underbrace{T_0}_{0} = A_{0,1}(m\vec{g})$$

$$\frac{1}{2}\cancel{m} V_1^2 = mgh = 2\cancel{m}g$$

$$\boxed{V_1^2 = 4g}$$

$$m\vec{a} = m\vec{g} + \vec{N}$$

$$n:\ m\frac{V_1^2}{R_{K1}} = N_1 - mg \Rightarrow N_1 = mg + m\frac{V_1^2}{R_{K1}}$$

$$N_1 = m\left(g + \frac{V_1^2}{R_K}\right) = mg\left(1 + \frac{4}{R_{K1}}\right)$$

$$R_{kr1} = \frac{(1+y_1'^2)^{3/2}}{|y_1''|} = \frac{1}{2}$$

$$y_1' = 2x_1 = 0$$

$$y_1'' = 2$$

$$\boxed{N_1 = 0{,}01 \cdot 9{,}81 \left(1 + \frac{4}{\frac{1}{2}}\right) = 0{,}0981 \cdot 9 \approx 0{,}8829 N}$$

3.30

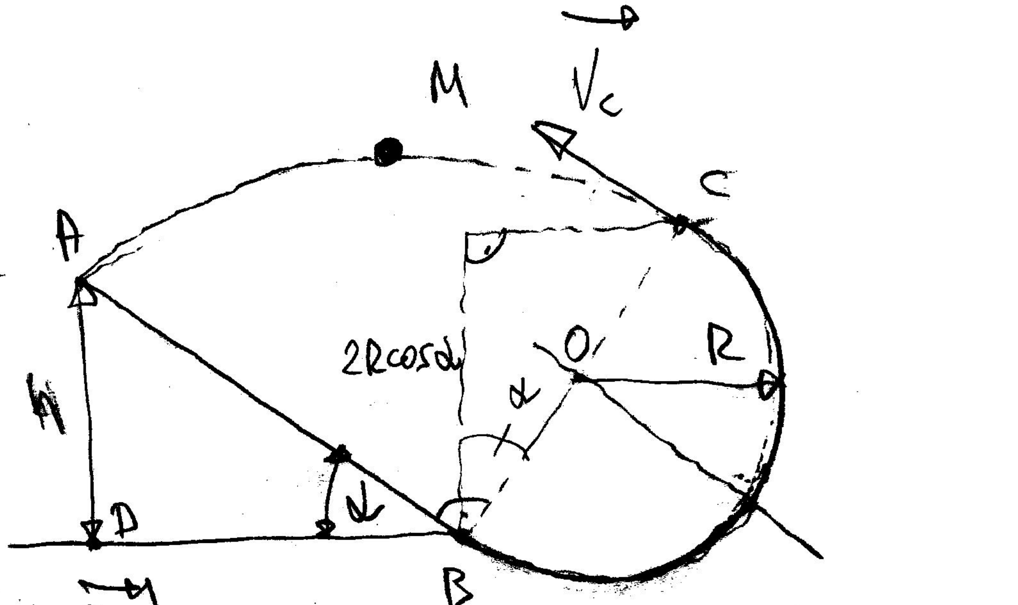
$\overline{DA} = h$

$\alpha = 30^\circ$

m

$R = h\sqrt{3}$

$V_0, N_C - ?$



$$T_C - T_A = A_{A,C}(m\vec{g})$$

$$\frac{1}{2}mV_C^2 - \frac{1}{2}mV_0^2 = mg(h - 2R\cos\alpha) \;/\cdot\frac{2}{m}$$

$$V_C^2 = V_0^2 + 2g\left(h - \cancel{2}h\sqrt{3}\,\frac{\sqrt{3}}{\cancel{2}}\right) = V_0^2 - 4gh$$

НАСТАВАК ЈЕ ЧСТИ (МЕХАНИКА 2 – ДИНАМИКА ТАЧКЕ)

СТР: 5–6